British Combinatorial Newsletter No. 9 (September 2010)

Remember this Newsletter aims to complement the Bulletin with some additional information about (e.g.) details of forthcoming meetings, summaries of recent movements of people, visitors, etc: records of “outreach” activities or recent breakthrough results in the subject: it might include a combinatorial problem or an occasional oddity. British Combinatorial Newsletters are produced at the start of the academic year (when the movements information is most useful to e.g. seminar organisers) and also around the time of the Bulletin (in April) to let you know what is coming up over the Summer. They are on the BCB website at http://www.essex.ac.uk/maths/BCB/newsletters.htm

If you have material which you think might be suitable for inclusion, or suggestions as to how the newsletter should evolve, please contact the Editor, David Penman (dbpenman@essex.ac.uk). The Editor reserves control of content.

Forthcoming regular meetings supported by the BCC.

PCC: The most recent meeting took place from 3-7 July 2010, at QMUL, organised by Andrew Drizen and John Faben. The venue for the next meeting will be announced in due course.

Old Codgers’ Meeting: The next such meeting will take place at Reading on Wednesday 3 November 2010. The meeting will partly be in memory of David Daykin who sadly died recently. The speakers will include Rudolf Ahlswede, Peter Cameron, Jackie Daykin, Anthony Hilton and Curt Lindner. More details will be forthcoming shortly.

Open University Winter Combinatorics Meeting: The next Open University Winter Combinatorics meeting will take place on Wednesday 2 February 2011. More details will appear in due course at http://wcm.open.ac.uk/ which also includes information about the earlier meetings in the series.

Oxford 1-day meeting in Combinatorics: The most recent Oxford meeting was on Wednesday 17 March 2010, details at http://people.maths.ox.ac.uk/~scott/ Again, it is hoped there will be a similar meeting at roughly the same time in 2011.

London: The most recent linked London meetings were on 19th and 20th May 2010 http://www2.lse.ac.uk/maths/Seminars/Colloquia_in_Combinatorics_Write-up.asp It is hoped there will be a similar meeting at roughly the same time in 2011.

BCC2011: The next (2011) BCC (the 23rd) will be at Exeter from 3-8 July 2011. (Note these dates are somewhat earlier than was initially informally suggested). The Local Organiser is Robin Chapman. Further details will be announced in due course.
You are reminded that the Editor maintains a mailing list for advertising other forthcoming UK meetings, Ph.D student level or above courses, etc. in combinatorics (broadly interpreted). Please email him if you would like to publicise such a meeting. Remember lists of forthcoming conferences in Combinatorics and related areas can be found at http://www.maths.qmul.ac.uk/~pjc/bcc/conferences.html or http://www.math.uiuc.edu/~west/meetlist.html

Movements.

**Birkbeck:** Dr. Amarpreet Rattan, formerly a Research Fellow at Bristol, is now a Lecturer at Birkbeck. He is interested in algebraic and combinatorial enumeration. New website http://www.ems.bbk.ac.uk/faculty/rattan/index_html, old website http://www.maths.bris.ac.uk/~maxar/

**Birmingham:** Dr. Diana Piguet, currently a DIMAP Research Fellow in Warwick, will be joining the Department in Birmingham from October 2010. She is interested in extremal graph theory and Ramsey theory: http://www2.warwick.ac.uk/fac/sci/maths/people/staff/diana_piguet/

**Bristol:** In addition to Dr. Amarpreet Rattan leaving (see above) and Dr. Robert Brignall’s previously indicated move to the OU, Dr. Jeanette McLeod will be leaving Bristol at the end of October.

**Glasgow:** Dr. Rob Irving, formerly a Senior Lecturer at Glasgow, is now an Honorary Research Fellow there. Dr. Peter Biro (formerly Research Associate) has left.

**Royal Holloway:** Dr. Mark Wildon, formerly a research fellow at Bristol, has now taken up a Lectureship: he is interested in representation theory and combinatorics, see http://personal.rhul.ac.uk/uvah/099/ Dr. Christian Elsholtz left at the end of April to take up a post at the Technisches Universitat Graz, Austria. Prof. Peter Wild will retire at the end of September (and probably return to Australia soon after that).

**St. Andrews:** Dr. Collin Bleak, formerly at the University of Nebraska at Lincoln, is now a researcher at St. Andrews. He is interested in geometric group theory and automatic structures. Old homepage http://www.math.unl.edu/~cbleak2/, new one http://www.mcs.st-andrews.ac.uk/homepages/cb211.html

**UCL:** Dr. Maria Prodromou, former Research Fellow, has left.

**Warwick:** Dr. Peter Allen, former DIMAP Research Fellow, has left to take up a post in Sao Paolo. Dr. Diana Piguet will shortly be moving to Birmingham (see above).

Recent Ph.D. theses in Combinatorics.
Again, not more accurate than the information I receive: “recent” may be ill-defined.

**Birmingham:** Richard Mycroft defended his Ph.D thesis on “The Regularity Method in Graphs and Hypergraphs” in August 2010. Deryk Osthus was his supervisor.

**Durham:** Pim van ‘t Hof defended his Ph.D thesis on “Exploiting structure to cope with NP-hard graph problems: Polynomial and exponential time exact algorithms” in May 2010. Daniel Paulusma was the supervisor.

**UCL:** Selvinaz Sezgin has been awarded a Ph.D. for a thesis on “The Unrestricted Blocking Number in Convex Geometry”: [http://eprints.ucl.ac.uk/19509/1/19509.pdf](http://eprints.ucl.ac.uk/19509/1/19509.pdf) David Larman was her supervisor.

### New Courses etc.

**Kent:** Have recently created an M.Sc. in Mathematics and its Applications. Modules include “Poisson algebras and Combinatorics” and “Symmetries, Groups and Invariants.”

### Landscape Document

Peter Cameron is currently writing the Landscape Document for the International Review of Mathematics in the UK which will take place in December 2010. Details of the process, the panel membership, etc. may be found at [http://www.epsrc.ac.uk/research/intrevs/2010maths/Pages/default.aspx](http://www.epsrc.ac.uk/research/intrevs/2010maths/Pages/default.aspx) and about the information that Peter wants at [http://www.maths.qmul.ac.uk/~pjc/landscape/](http://www.maths.qmul.ac.uk/~pjc/landscape/)

Anyone who has not yet sent information to him ([p.j.cameron@qmul.ac.uk](mailto:p.j.cameron@qmul.ac.uk)) but wants to should do so **as soon as possible** and certainly by the end of September when the process has to be finished.

### (Unsolved) Problem(s)

This is an old chestnut (and a bonus problem) which many people react to on first hearing it by guessing it must be simple, e.g. an exercise in the Sylow theorems. It appears at present to be far from simple!

**Problem.** Let $G$ be a transitive group of permutations of a finite set $\Omega$, where $|\Omega| > 1$. Prove directly and briefly (and in particular without using the Classification of Finite Simple Groups!) that $G$ contains a derangement (i.e. a fixed-point-free permutation) which has prime power order.

**Context:** The fact that $G$ contains a derangement is a trivial consequence of the Orbit-Counting Lemma (aka Burnside’s Lemma, aka Not Burnside’s Lemma. Mathematicians are very logical), since it says that in a transitive group the average number of fixed points of an element is 1: the identity fixes strictly more than one point,
thus some element fixes strictly fewer than 1 points, i.e. is a derangement. This fact, due to Jordan in 1871, though easy, is already useful: see Serre’s paper at
for applications in number theory and topology, and for another application to curves over finite fields see Guralnick and Wan’s
http://www.math.uci.edu/~dwan/bob.ps

In fact, Cameron and Cohen showed that a proportion at least \((r - 1)/|\Omega| \geq 1/|\Omega|\)
of the elements of \(G\) are derangements, where \(r\) is the number of orbits of \(G\) on ordered pairs from \(\Omega\) (the rank). See http://oai.cwi.nl/oai/asset/203/0203A.pdf for the proof and related discussion: see also http://www.math.wisc.edu/~boston/ fixed.pdf by Boston et al. including e.g. a proof that if \(G\) is nilpotent, at least half the elements are derangements. In http://www-bcf.usc.edu/~fulman/dfgsubmit.pdf Diaconis, Fulman and Guralnick give an upper bound \(1 - \frac{1}{r}\) on the proportion of derangements (the bound stated in their paper is slightly different, but easily upgraded to this one, which is sharp for regular permutation groups), and several other interesting facts about derangements and the contexts in which they crop up.

The proof of existence of a derangement is unconstructive and gives no idea of what kinds of elements are derangements. (Can one get this problem into a form where you can apply derandomization ideas to it?)

In 1981 Fein, Kantor and Schacher proved that there is indeed a derangement of prime power order in a transitive permutation group. However their proof relies on the classification of finite simple groups (and non-trivial work beyond that). The possibility that there might be a more straightforward proof remains open. Their result has applications in number theory too, namely that the relative Brauer group of any finite extension of global fields is infinite (these terms are defined in e.g. Section 6.6. of Peter Cameron’s book on permutation groups).

Fein, Kantor and Schacher also muddied the waters by noting that there is not always a derangement of prime order – examples include the Mathieu group \(M_{11}\) in its action on twelve elements, or the action of the 1-dimensional affine group over the finite field of order 9 acting on the 12 lines of the affine plane of order 3. Groups which do not have a derangement of prime order are called elusive, and we know an increasing amount about them - see e.g. http://school.maths.uwa.edu.au/~giudici/alsporadfpf.pdf by Burness, Giudici and Wilson, and references therein for recent results in this area.

Another area is to ask for which prime(s) there is a prime-power order derangement. It is plausible that if a very large power of a prime \(p\) divides the order of \(\Omega\), then there should be a derangement of order \(p\). This was conjectured by Isbell (for \(p = 2\)) in the context of game theory. The problem seems to be open, though a slightly stronger conjecture by Cameron in this area has been refuted by Crestani and Spiga.

If all this is too much for you, you might find the following problem of John Thompson, which has resurfaced in various recent talks by Tim Burness, easier. Or not... (The definition of primitive is in all books on permutation groups: it is equivalent to transitive plus all the point stabilisers are maximal subgroups of \(G\).

**Bonus Problem.** Let \(G\) be a primitive group of permutations of a set \(\Omega\), where \(|\Omega| > 1\). Is the set of derangements (which of course is never a group) transitive on \(\Omega\)?

It is apparently known that the primitivity requirement really is needed, that the answer is “yes” if additionally the group is 2-transitive, and that it suffices to prove it for “almost simple” groups (i.e. groups \(G\) for which there exists a non-abelian simple group \(S\) such that \(S \leq G \leq \text{Aut}(S)\)). No counterexample is known.