British Combinatorial Newsletter No. 21 (September 2016)

Remember this Newsletter aims to complement the Bulletin with some additional information about (e.g.) details of forthcoming meetings, summaries of recent movements of people, visitors, etc: records of “outreach” activities or recent breakthrough results in the subject: it might include a combinatorial problem or an occasional oddity. British Combinatorial Newsletters are produced at the start of the academic year (when the movements information is most useful to e.g. seminar organisers) and also at around the time of the Bulletin (end of April or so) to let you know what is coming up over the Summer. They are on the BCB website at http://www.essex.ac.uk/maths/BCB/newsletters.htm

If you have material which you think might be suitable for inclusion, or suggestions as to how the newsletter should evolve, please contact the Editor, David Penman (dbpenman@essex.ac.uk). The Editor reserves control of content.

Forthcoming meetings

The next 2015 BCC (the 26th) will be at Strathclyde. The dates are 3 - 7 July 2017, and the Local Organiser is Segey Kitaev, with other organisers David Bevan (Strathclyde), Anders Claesson (Iceland), David Manlove (Glasgow), Kitty Meeks (Glasgow) and Einar Steingrímsson (Strathclyde)

The website is http://combinatorics.cis.strath.ac.uk/bcc2017/ which contains information on invited speakers, venue, excursion, etc. Further details on registration etc. will be announced in due course.

Regular short meetings supported by the BCC:

Old Codgers meeting at Reading. The next such meeting is down for Wednesday 2 November 2016 at Reading (and not any other day which may have been previously advertised in some places). Speakers will be Peter Cameron (St. Andrews) on "Some of my favourite combinatorial problems", David Conlon (Oxford) on "Two themes in quasirandomness", Terry Griggs (Open) on "Pentagonal Geometries", Matthew Johnson (Durham) on "Kempe equivalence of regular graphs", Robert Johnson (QMUL) (topic TBA) and Steven Noble (Birkbeck) on "Graphs are to matroids what embedded graphs are to????". More details will be announced in due course.

Open University Winter Combinatorics Meeting. The most recent meeting took place on 20 January 2016, details at http://wcm.open.ac.uk/ where it is hoped details of the next meeting will be posted in due course. However there will not be an OU meeting in 2017 owing to the other two meetings at Royal Holloway and Durham in January 2017, see below.

Scottish Combinatorics Meeting. The second in this series took place
April 2016. The webpage was http://www.maths.gla.ac.uk/~kmeeks/scm/ and it is hoped there will be a similar meeting next year.

**London:** The next pair of linked one-day London colloquia in Combinatorics are on 10 and 11 May 2017. Details will in due course be available at http://www.lse.ac.uk/maths/Seminars/Colloquia_2017.aspx

**Oxford 1-day meeting in Combinatorics:** The most recent meeting took place on Wednesday 1 June 2016. Details are at http://people.maths.ox.ac.uk/scott/Pages/one-day_meeting.htm where it is hoped details of the next meeting will take place in due course.

**Postgraduate Combinatorial Conference (PCC):** The next PCC will be being organised by Anja Komatar (Leeds). Details will be announced in due course.

You are reminded that the Editor maintains a mailing list for advertising other forthcoming UK meetings, Ph.D. student level or above courses, etc. in combinatorics (broadly interpreted). Please email him if you would like to publicise such a meeting. Remember lists of forthcoming conferences in Combinatorics and related areas can be found at http://www.maths.qmul.ac.uk/~pjc/bcc/conferences.html or http://www.math.uiuc.edu/~west/meetlist.html

**Other Forthcoming Conferences and Meetings.**

Other interesting UK meetings and/or courses are coming up:

**Gutin60.** Gregory Gutin’s 60 birthday conference will take place at Royal Holloway on the 7th and 8th of January 2017. The invited speakers are Noga Alon, Jørgen Bang-Jensen, Fedor Fomin, Mark Jones, Daniel Karapetyan, Eunjung Kim, Michael Krivelevich, Igor Razgon, Saket Saurabh, Benny Sudakov, Stefan Szeider and Anders Yeo. More information can be found at http://gutin60.ma.rhul.ac.uk which contains details of the schedule and how to get to RHUL and some information on accommodation. Registration is free but the organisers want to know in advance who is coming so that refreshments can be planned.

**Algebraic, Topological and Complexity Aspects of Graph Covers 2017.** This will take place at Durham from 9-13 January 2017. The webpage is http://community.dur.ac.uk/atcagc.2017/ with main speakers Pavol Hell (Simon Fraser), Dan Kráľ’ (Warwick) and Edita Mácajová (Comenius). Early bird registration until 1st November and closing date for registration is 16 December. There may be limited possibilities for contributing a talk and details of how to do this, plus information on accommodation, are on the website.

**Movements.**
**Birkbeck.** Dr. Steven Noble, formerly at Brunel, has taken up a Readership at Birkbeck.

**Birmingham:** Dr. Andrew McDowell has left to do a postdoc at Kings College London with Colin Cooper. Dr. Mykhaylo Tyomkyn moved to Tel Aviv to work with Asaf Shapira in February.

**Dundee:** Dr. Saša Radomirović (Combinatorics on words, cryptographic protocols, information security) is now a Senior Lecturer in Computing. Webpage [http://www.computing.dundee.ac.uk/about/staff/146](http://www.computing.dundee.ac.uk/about/staff/146)

**Glasgow:** Dr Kitty Meeks (University of Glasgow) has been awarded a 5-year Personal Research Fellowship by the Royal Society of Edinburgh. She will move from the School of Mathematics and Statistics to the School of Computing Science for the Fellowship and on its completion will become a Lecturer in Computing Science.

**Plymouth:** Dr. Tom McCourt has left Plymouth to return to a post in Australia.

**Strathclyde:** In addition to his Fellowship at the Open University, Dr. David Bevan is now a lecturer in the Combinatorics Group at Strathclyde.

**Unsolved Problem**

This arises from a recent paper of Seongmin Ok and Carsten Thomassen, to appear in Journal of Graph Theory – [http://onlinelibrary.wiley.com/doi/10.1002/jgt.22026/epdf](http://onlinelibrary.wiley.com/doi/10.1002/jgt.22026/epdf). The article is about, for a multigraph on \(n\) vertices \(G\) (we allow multiple edges, but do not allow loops) the minimum number of spanning trees, when we impose the additional condition of edge-connectivity \(k\). The multigraphs can have non-trivially fewer spanning trees than in the corresponding question for simple graphs, where (for example) Sasha Kostochka showed that for a \(k\)-regular simple graph we have that 

\[
\tau(G)^{1/n} \text{ is bounded above by } k \text{ (exercise!)} \text{ and below by a function of } k \text{ which is (more precise than) } k(1 - o(1)) \text{ for large } k.
\]

The “vanilla” lower bound in Ok and Thomassen’s paper for \(k\)-edge-connected multigraphs is \(n(k/2)^{n-1}\). This vanilla bound is tight for even \(k\) (take an \(n\)-cycle in which every edge has multiplicity \(k/2\)). However this bound is not optimal if \(k\) is odd, where there are a sequence of lower bounds \(c(k)^{n-1}\) with \(c(k) \geq k^2/2\) for example 

\[
c(3) < 1.77. \ c(3) \text{ is also less than } \sqrt{2} + \sqrt{3} \approx 1.93. \ 1.93 \text{ is less than } 4/2 \text{ (exercise….. so, in accordance with intuition, } c(3) \text{ is less than the corresponding number } c(4) \text{ for 4-regular graphs from the vanilla lower bound.}
\]

This is the point where peculiar things start happening. For example, the following statements appear currently to be true:

1. There do not appear to be any known examples of 5-edge-connected multigraphs with fewer spanning trees than the \(n\)-cycle with all edges of multiplicity 3 except for one, which is almost 6-regular
2. There do not appear to be any known examples of 5-regular 5-edge connected graphs with fewer than 3.09\(^{n-1}\) spanning trees - 3.09 is of course greater than \(c(6) = 3\) from the vanilla theorem and its tightness.
3. The oddity analogous to 2 above happens for all $k = 2r - 1 \geq 5$. Indeed there is, for any odd number $k$, a graph, the so-called multiprism, with $n = 2t$ vertices, $MP_{2t}(k)$ which is $k$-regular and $k$-edge-connected with (asymptotically) $((k + 2)/2)^n$ spanning trees – which is more than the number of spanning trees in the $n$-cycle with all edges of multiplicity $(k + 1)$.

**Problem:** Explain all of this!

The problem is worded in a deliberately open-ended way. Several other more specific open problems are stated in the paper.